1. Fresnel Reflection from an Interface

Please explain the following facts concerning linearly polarized beams reflected from a planar interface between two homogeneous media of different refractive indices, n_i and n_t. θ_i is the incident angle to the surface normal. $R_\perp$ and $R_{||}$, respectively, refer to the reflectances of beams for which the electric field vectors $\vec{E}$ are perpendicular and within the plane of incidence. Provide sketches as needed.

(a) Which physical properties of the media determine the magnitude of $R_\perp$ and $R_{||}$ at normal incidence ($\theta_i = 0°$)? **(4 pts)**

Just n_i and n_t.

[$R_\perp$ ($= r_\perp^2$) $= R_{||}$ ($= r_{||}^2$) $= (n_\mathrm{i} - n_\mathrm{t})^2/(n_\mathrm{i} + n_\mathrm{t})^2$; not explicitly required for answer]

(b) Why is $R_\perp = R_{||}$ at normal incidence ($\theta_i = 0°$)? **(4 pts)**

$R_\perp$ and $R_{||}$ refer to the orientation of $\vec{E}$ with respect to the Plane of Incidence. The PoI is not defined at normal incidence.

(c) Why is $R_\perp = R_{||} = 1$ at grazing incidence ($\theta_i = 90°$)? **(4 pts)**

Various correct answers possible:

– At grazing incidence, the incident beam doesn't penetrate media and therefore doesn't excite scatterers.

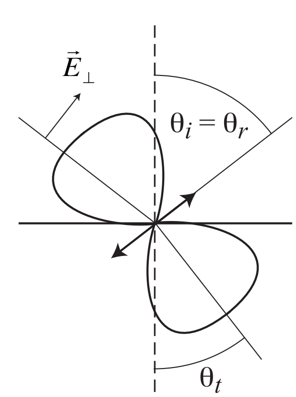
– The boundary conditions for $\vec{E}$ and $\vec{B}$ are satisfied with the incident and reflected beams alone

$\rightarrow t_\perp = t_{||} = 0$

– $r_\perp$ and $r_{||} \rightarrow \pm 1$, as $\cos\theta_i = 0$ (for $n_\mathrm{i} < n_\mathrm{t}$) or $\cos\theta_t = 0$ (for $n_\mathrm{i} > n_\mathrm{t}$)

(d) Why is there a specific angle – the polarization angle θ_p – where $R_{||} = 0$? **(8 pts)**

$\theta_i = \theta_p$ occurs for $\vec{E}_\perp$ when $\theta_i + \theta_t = 90°$. That direction coincides with the symmetry axis of the dipolar scatterers where the emission intensity is zero.



2. Conjugate Points of an Optical System

Consider a positive (converging) thin lens of focal length, f, placed between an object and an image board, which are separated by a fixed distance, L.

(a) Show that for $L > 4f$ there are two different locations of the lens (two values of the distance s_0 between the object and the lens), which will give a well-focused image. Express the two values of s_0 in terms of L and f. **(20 pts)**

Let the image distance to the lens be s_i.

$$\frac{1}{s_0}+\frac{1}{s_i}=\frac{1}{f} \quad \rightarrow \quad \frac{1}{s_i}=\frac{1}{f}-\frac{1}{s_0}=\frac{s_0-f}{s_0 f}$$

$$s_i=\frac{s_0 f}{s_0-f}; \quad L=s_0+\frac{s_0-f}{s_0 f}=\frac{s_0^2}{s_0-f}$$

$$s_0^2-Ls_0+Lf=0 \quad \rightarrow \quad s_0^{\pm}=\frac{L\pm\sqrt{L^2-4Lf}}{2}$$

(b) Show for $L > 4f$ that these two possible locations of the lens are separated by a distance d given by the expression

$d^2 = L^2 - 4Lf$ **(10 pts)**

$$d=s_0^{+}-s_0^{-}=\sqrt{L^2-4Lf} \rightarrow d^2=L^2-4Lf$$

3. Dispersion Relation (20 pts)

In a system of forced molecular oscillators, N is their volume density, q_e is their electronic charge and m_e their mass. ω_0 denotes the angular resonance frequency ($\omega_0^2 = k_m / m_e$, where k_m is the linear force constant of a restoring force). Show that the dispersion relation for the resulting index of refraction, n,

$$n^2 = 1 + \frac{Nq_e^2}{\varepsilon_0 m_e \left(\omega_0^2 - \omega^2\right)}$$

can be rewritten as $C\left(\frac{1}{\lambda_0^2} - \frac{1}{\lambda^2}\right)$ and determine C.

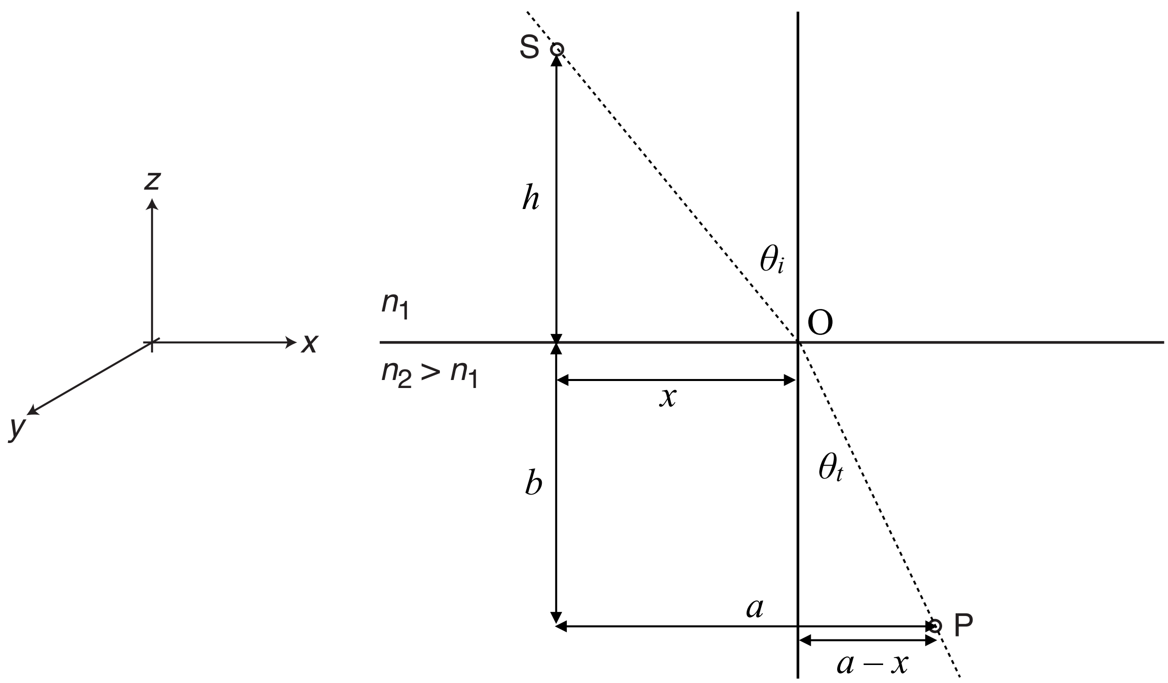
$n^2 = 1 + \mathrm{C}^*/(\omega_0^2 - \omega^2)$ with $\mathrm{C}^* = Nq_e^2/(\varepsilon_0 \cdot m_e)$

$$\rightarrow 1/(n^2 - 1) = (\omega_0^2 - \omega^2)/\mathrm{C}^* = \frac{4\pi^2 c^2}{\mathrm{C}^*}\left(\frac{1}{\lambda_0^2} - \frac{1}{\lambda^2}\right) = \mathrm{C}\left(\frac{1}{\lambda_0^2} - \frac{1}{\lambda^2}\right)$$

$$\rightarrow \mathrm{C} = \frac{4\pi^2 c^2 \varepsilon_0 m_e}{Nq_e^2}$$

4. Fermat's Principle and Snell's Law (30 pts)

To derive Snell's Law by using the Fermat Principle, determine that light path between a source point S and a light detector P that minimizes the flight time for the situation depicted below. Draw an approximate path geometry in the sketch, label the relevant geometrical quantities, and then determine the actual path analytically by minimizing the flight time.



Fermat: minimize flight time between S and P

$$t = \frac{\overline{SO}}{v_1} + \frac{\overline{OP}}{v_2}; \quad v_i = c/n_i$$

$$t = \frac{\sqrt{h^2 + x^2}}{v_1} + \frac{\sqrt{(a-x)^2 + b^2}}{v2} = \frac{1}{c}\left(n_1\sqrt{h^2 + x^2} + n_2\sqrt{(a-x)^2 + b^2}\right)$$

$$\frac{\mathrm{d}t}{\mathrm{d}x} = 0 = \frac{1}{c}\left(\frac{n_1 x}{\sqrt{h^2 + x^2}} - \frac{n_2(a-x)}{\sqrt{(a-x)^2 + b^2}}\right) \text{ or } \frac{n_1 x}{\sqrt{h^2 + x^2}} = \frac{n_2(a-x)}{\sqrt{(a-x)^2 + b^2}}$$

$\rightarrow n_1 \sin\theta_i = n_2 \sin\theta_t$, *i.e.*, Snell's law